Solving Combinatorial Optimization Problems by Nonlinear Neural Dynamics

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ABSTRACT

The new approach for combinatorial optimization problems using chaotic dynamics is discussed. We show effectiveness of chaotic neuro dynamics for solving combinatorial optimization problems by applying the chaotic neural network to traveling salesman problems. In this paper, we adopt the chaotic neural network model with two internal states, corresponding to mutual interactions which minimize an energy function and refractoriness which induce chaotic dynamics. We investigate relationships between solving abilities and different model parameters such as decay parameters of two internal states, Lyapunov exponents and first order statistics of firing patterns.

1. Introduction

The effectiveness of chaotic dynamics on solving combinatorial optimization problems has been studied by some researchers. Nozawa modified the mutual connection neural network [1] by Euler's method to the neural network possessing the self feedback connections, which is the same as the chaotic neural network [2, 3], and applied it to the traveling salesman problems (TSP) [4]. Yamada et al. investigated the solving ability of TSP with chaotic neural networks [2, 3] by comparing to the neural network model with stochastic dynamics. They suggested that chaotic dynamics are more effective for solving optimization problems, such as TSP, than stochastic dynamics [5]. They also implied that dynamics of chaotic neural networks near to "the edge of chaos," at which Lyapunov exponents are almost 0, exhibits higher solving abilities[6] and network dynamics becomes more complicated [7, 8]. The effectiveness of chaotic simulated annealing was also shown by Chen et al.[9] with application to TSP and maintenance scheduling problems in a power system.

As for another applications of chaotic dynamics, associative memories have been discussed and it has been shown that chaotic neural network can have various associative dynamics by exploring parameter spaces[10, 11, 12]. In these researches, chaotic neural networks with two internal states (2-CNN) are used: they express refractoriness of neurons and mutual interactions between neurons. It has been clarified that dynamical retrieval process can be realized with 2-CNN. Since the chaotic dynamics are mainly produced by the refractoriness [2, 13], effective dynamical search is realized by setting refractoriness relatively larger than mutual interactions. When solving TSP, many undesirable local minima and the global minimum can be considered as the stored patterns of associative memory. Since the optimal solution of TSP is stored as the global minimum in a state space, dynamical search may be

effective as is the case of associative memory. However, in solving TSP, essential requirement is not only the dynamical search, but the minimization of the total tour length. In the previous researches for solving combinatorial optimization problems using chaotic dynamics, a simple version of chaotic neural networks, which has single internal state (1-CNN), has been used. It is natural to expect that TSP may be solved more effectively by using 2-CNN and considering the balance of two kind of dynamics: namely, descent down hill dynamics due to mutual interactions and the chaotic dynamics due to refractoriness. Therefore, in this paper, we investigate the effectiveness of chaotic dynamics by applying 2-CNN to combinatorial optimization problems. We examine the relation between parameters of two internal states and solving abilities of TSP by 2-CNN. As a measure of chaotic dynamics, we calculate the Lyapunov exponents of chaotic neural networks and compared them with solving abilities in case of changing control parameters for mutual interactions and refractoriness. Furthermore we calculated a kind of first order statistics of the firing sequences of chaotic neural networks, that is, standard deviations normalized by averages of firing intervals, as one of a measure of the neural network dynamics, and compared it with the solving abilities.

2. Chaotic neural network model with two kinds of internal states[2,3].

Chaotic neural network can produce various response, including not only periodic but also chaotic ones, by changing its parameters [2, 3]. The dynamics of i th chaotic neuron in a chaotic neural network is defined as follow:

$$\begin{aligned} x_i(t+1) = f_i[&\sum_{j=1}^{N_A} V_{ij} \sum_{d=0}^{t} k_s^d A_j(t-d) \\ &+ \sum_{j=1}^{N_x} W_{ij} \sum_{d=0}^{t} k_m^d x_j(t-d) \\ &- \alpha \sum_{d=0}^{t} k_r^d x_i(t-d) - \theta_i] \end{aligned} \quad (1)$$

where $x_i(t+1)$ is the output of the i th chaotic neuron at discrete time $t+1$; $A_j(t)$ is the strength of the jth input externally applied at the discrete time t and N_A is the number of the externally applied inputs; V_{ij} is the connection weight from the jth externally applied input to the i th chaotic neuron; W_{ij} is the connection weight from the j th chaotic neuron to the i th chaotic neuron; f_i is the continuous output function of the i th chaotic neuron; a_i is the threshold of the i th chaotic neuron; k_s,k_m and k_r are the decay parameter for external inputs, feedback inputs and refractoriness, respectively[2].

Defining three internal states corresponding to externally applied inputs, mutual interactions and refractoriness by ξ_i, η_i and ζ_i as follows:

$$\xi_i(t+1) = \sum_{j=1}^{N_A} V_{ij} \sum_{d=0}^{t} k_s^d A_j(t-d), \quad (2)$$

$$\eta_i(t+1) = \sum_{j=1}^{N_x} W_{ij} \sum_{d=0}^{t} k_m^d x_j(t-d), \quad (3)$$

$$\zeta_i(t+1) = -\alpha \sum_{d=0}^{t} k_r^d x_i(t-d) - \theta_i, \quad (4)$$

$$x_i(t+1) = f_i[\xi_i(t+1) + \eta_i(t+1) + \zeta_i(t+1)] \quad (5)$$

Then, we obtain a reduced form of chaotic neural networks shown in the following equations:

$$\xi_i(t+1) = k_s \xi i(t) + \sum_{j=1}^{N_A} V_{ij} A_j(t), \quad (6)$$

$$\eta_i(t+1) = k_m \eta_i(t) + \sum_{j=1}^{N_x} W_{ij} x_j(t), \quad (7)$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha x_i(t) + a_i, \quad (8)$$

$$x_i(t+1) = f_i[\eta_i(t+1) + \zeta_i(t+1)], \quad (9)$$

where $a_i = -\theta_i(1-k_r)$. For the application to TSP, the external inputs are not required, therefore Eqs. (6)–(9) can be reduced to Eqs. (10)–(12) [10, 11, 12]. Then, the dynamics of i th chaotic neuron in chaotic neural networks with two internal states is described as follows:

$$\eta_i(t+1) = k_m \eta_i(t) + \sum_{j=1}^{N_x} W_{ij} x_j(t), \quad (10)$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha x_i(t) + a_i, \quad (11)$$

$$x_i(t+1) = f_i[\eta_i(t+1) + \zeta_i(t+1)], \quad (12)$$

where f_i is a sigmoidal function, $f_i(z) = 1/(1 + \exp(-z/\epsilon))$. If the parameter values k_m and k_r are set to the same value, the model of 2-CNN is reduced to the model of 1-CNN [2, 3].

3. An application of chaotic neural networks to the Traveling Salesman Problem.

The method for solving TSP using an energy function of a neural network was originally proposed by Hopfield and Tank[1]. They show that the states of a neural network converges to a stable equilibrium point corresponding to the minimum value of an energy function. We adopt the formulation for TSP proposed by Hopfield and Tank [1]. Therefore in order to solve TSP with N cities, $N \times N$ neurons

are prepared, and they are arranged in the $N \times N$ grid. The firing of (i, j) th neuron means that the i th city is visited on the j th order. With this neural networks, the following quasi-energy function is calculated[4],

$$E_{tsp}=A[\{\sum_{i=1}^{N}(\sum_{k=1}^{N}\bar{x}_{ik}(t)-R)\}^2+\{\sum_{k=1}^{N}(\sum_{i=1}^{N}\bar{x}_{ik}(t)-R)\}^2] + B\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}d_{ij}\bar{x}_{ik}(t)\{\bar{x}_{jk+1}(t)+\bar{x}_{jk-1}(t)\} \quad (13)$$

where A and B are positive constants, d_{ij} is the distance between i th and j th city, R is a parameter for controlling the average firing rate, and $\bar{x}_{ik}(t)$ is the short term average firing rate defined by [4, 6],

$$\bar{x}_{ik}(t) = \frac{1}{\tau}\sum_{l=t-\tau+1}^{t} x_{ik}(l). \quad (14)$$

From Eq. (13), weights of synaptic connections between the (i, k) th neuron and the (j, l) th neuron and thresholds of the (i, k) th neuron are calculated as follows[4],

$$w_{ikjl} = -A\{\delta_{ij}(1-\delta_{kl})+\delta_{kl}(1-\delta_{ij})\} - Bd_{ij}(\delta_{lk+1}+\delta_{lk-1}) \quad (15)$$

$$\theta_{ik} = 2AR \quad (16)$$

where δ_{ij} denotes Kronecker's delta. We assumed that the firing of a neuron is "1", and the resting "0", then the conditions of firing and resting of the (i, j) th neuron at time t, $\hat{x}_{ij}(t)$, is denoted as follow:

$$\hat{x}_{ij}(t) = \begin{cases} 1 \ldots \bar{x}_{ij}(t) \geq \tilde{x}(t), \\ 0 \ldots \bar{x}_{ij}(t) < \tilde{x}(t), \end{cases} \quad (17)$$

where $\tilde{x}(t)$ is the N th largest value of $\bar{x}_{ij}(t)$.

The results with the Hopfield's original data [1] are shown in Fig.1 (a) with $\alpha = 0.6$, $\epsilon = 0.125$, $R = 0.115$, $k_m = 0.725$, $\tau = 20$. The solving ability is defined as the rate of the appearance of the optimum solution in 1,000 iterations averaged on 500 different initial states.

It can be seen that the larger k_r leads to the higher solving abilities. However, if k_r is too large, the solving ability gets worse, for example, in case that k_r is larger than 0.95. We check searching dynamics of chaotic neural networks with these parameter spaces, and confirm that the response of chaotic neural networks with the lower k_r converge soon to a local minimum near to an initial condition and never change the states. We also find that dynamics with the larger k_r never converge but exhibit strong chaotic wonderings. It implies that the larger k_r leads to strong dynamical search, however it is not effective to use too strong dynamical search for solving TSP. The highest solving ability range is observed between the 0.87 and 0.9 of k_r. Here, parameter range of the best solving abilities can be considered as "the edge of chaos,"[6] and the range of k_r larger than 0.95 as "the disordered phase".

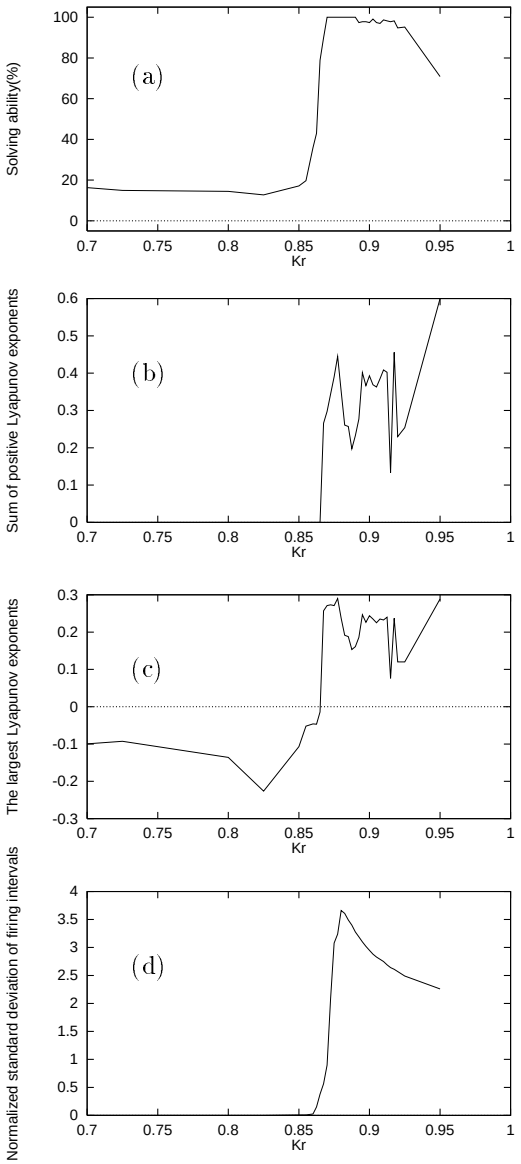

Fig. 1: (a)Solving abilities, (b)the sum of positive Lyapunov exponents,(c) the largest Lyapunov exponents and (d)normalized standard deviations of firing intervals, as a function of k_r. Solving abilities are averaged on 500 trials. The other parameters are described in the text.

Fig. 2 (a) shows the results in case of changing the value of k_m and fixing the value of k_r to 0.875 at which chaotic neural networks have the highest solving ability (Fig. 1 (a)). From this figure, if the value of k_m is too small, solving abilities are very low. It means that the information on the minimization of the total tour length is also important to obtain the optimal solution of TSP. We also check the dynamics of neural networks on these parameter spaces. We find that the response

of chaotic neural networks with the smaller k_m is dynamically fluctuating and never converge to some state, and the one with the large k_m converge to a local minimum.

From these two results, it is implied that high solving abilities of TSP can be realized by tuning k_r and k_m appropriately. With parameters used above, the parameter set of $k_r = k_m$ does not necessarily lead to high solving abilities. Although, the number of parameters of 2-CNN is larger than 1-CNN, it implies that it is better to solve TSP with 2-CNN than 1-CNN. Using 2-CNN makes it easy to change the balance between the refractoriness and minimization of an energy function, therefore more effective search with chaos neurodynamics can be realized.

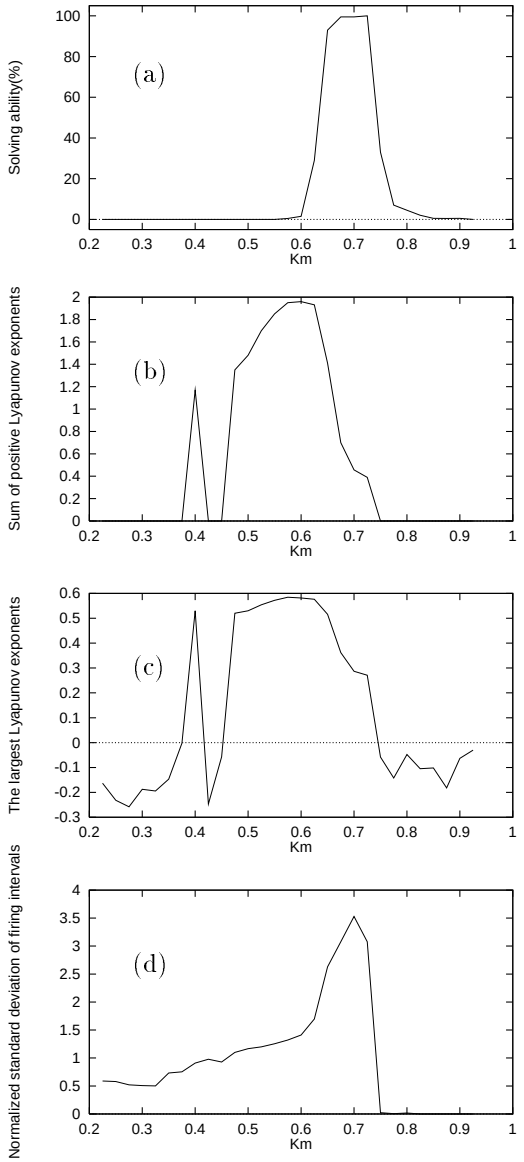

Fig. 2: The same as Fig.1 but as a function of k_m. The value of k_r is fixed to 0.875.

4. Lyapunov exponents of chaotic neural network.

In this section, we investigate how chaotic neural networks search the optimal solution on the high solving ability range quantitatively. To quantify chaotic dynamics, we calculate the Lyapunov spectra which characterize orbital instabilities of chaotic neural networks.

To obtain the Lyapunov spectra, first we have to calculate the Jacobian matrix. Since we use 2-CNN in this paper, the $2N^2$-dimensional internal state (N is the number of cities and 2 indicates two internal states.) $\boldsymbol{y}(t)$ is defined as,

$$\boldsymbol{y}(t) = (\eta_{11}(t), \ldots, \eta_{NN}(t), \zeta_{11}(t), \ldots, \zeta_{NN}(t)). \quad (18)$$

Then the Jacobian matrix of 2-CNN model is calculated as follow,

$$DF(\boldsymbol{y}(t)) = \begin{bmatrix} \frac{\partial \eta_{11}(t+1)}{\partial \eta_{11}(t)} & \cdots & \frac{\partial \eta_{11}(t+1)}{\partial \eta_{NN}(t)} & \frac{\partial \eta_{11}(t+1)}{\partial \zeta_{11}(t)} & \cdots & \frac{\partial \eta_{11}(t+1)}{\partial \zeta_{NN}(t)} \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \frac{\partial \eta_{NN}(t+1)}{\partial \eta_{11}(t)} & \cdots & \frac{\partial \eta_{NN}(t+1)}{\partial \eta_{NN}(t)} & \frac{\partial \eta_{NN}(t+1)}{\partial \zeta_{11}(t)} & \cdots & \frac{\partial \eta_{NN}(t+1)}{\partial \zeta_{NN}(t)} \\ \frac{\partial \zeta_{11}(t+1)}{\partial \eta_{11}(t)} & \cdots & \frac{\partial \zeta_{11}(t+1)}{\partial \eta_{NN}(t)} & \frac{\partial \zeta_{11}(t+1)}{\partial \zeta_{11}(t)} & \cdots & \frac{\partial \zeta_{11}(t+1)}{\partial \zeta_{NN}(t)} \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \frac{\partial \zeta_{NN}(t+1)}{\partial \eta_{11}(t)} & \cdots & \frac{\partial \zeta_{NN}(t+1)}{\partial \eta_{NN}(t)} & \frac{\partial \zeta_{NN}(t+1)}{\partial \zeta_{11}(t)} & \cdots & \frac{\partial \zeta_{NN}(t+1)}{\partial \zeta_{NN}(t)} \end{bmatrix}. \quad (19)$$

Lyapunov exponents are defined as follow,

$$\begin{aligned} \lambda_i &= \lim_{T \to \infty} \lambda_i(T) \\ &= \lim_{T \to \infty} \frac{1}{T} \log |\sigma_i(T)| \end{aligned} \quad (20)$$

where $\sigma_i(T)$ is an eigen value of the matrix DF_T, which is defined as follow,

$$DF_T = DF(\boldsymbol{y}(T-1))DF(\boldsymbol{y}(T-2)) \ldots DF(\boldsymbol{y}(0)). \quad (21)$$

In the numerical calculation, the Lyapunov spectra are estimated in $2N^2$- dimensional state space. In order to avoid singularities on numerical calculations, we use two methods of the orthogonalization, namely Gram-Schmidt procedure [14] and QR decomposition [15], however we cannot find any significant difference between both procedures. The calculation of Eq. (20) is stopped when the number of iterations T comes to be 50,000 or the convergence of Lyapunov spectra is successively observed, namely satisfying the condition that $|\lambda_1(T) - \lambda_1(T-1)| < 0.01$ for 1,000 times. Fig. 3 shows the convergence of the largest Lyapunov exponents with the two orthogonalization methods starting from six different initial states. From this figure, convergences on calculation of Lyapunov spectra are confirmed.

The results of calculating the Lyapunov spectra are shown in Figs.1 (b), (c), 2 (b), (c) ,4 and 5.

Figs. 1 (b) and 2 (b) show the summation of positive Lyapunov exponents, and Figs. 1 (c) and 2 (c) the largest Lyapunov exponent. In Fig.1 (b), the larger k_r leads to the larger Lyapunov exponents. The relations of the sum of positive Lyapunov exponents and the largest Lyapunov exponents to the solving abilities are shown in Figs.4 and 5, respectively. The solving abilities are low with non positive largest Lyapunov exponents which correspond to periodic dynamics (Figs. 2 (a), (b), (c), 4 and 5). The solving ability of 2-CNN model with using $k_m \approx 0.75$ are better than the abilities with the too small $k_m \approx 0.6$, however, the highest solving ability range does not necessarily correspond to the largest value of summation of positive Lyapunov exponents. It means that in this parameter range the dynamics is too chaotic for computing the optimal solution. From Fig.5, the high solving ability range is observed between the periodic and strong-chaotic ranges.

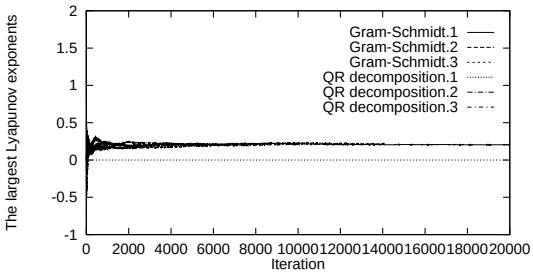

Fig. 3: The typical examples of the convergence of the largest Lyapunov exponents with two orthogonalization methods. $k_r = 0.8825, k_m = 0.725$.

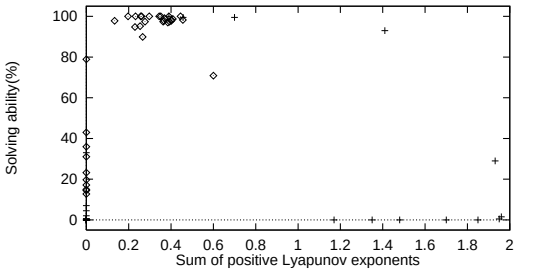

Fig. 4: The relations between solving abilities and the summation of positive Lyapunov exponents. Diamonds ($\diamond$) indicate the results of changing k_r, and plusses(+) indicate the ones of changing k_m. The expressions are the same for the following figures.

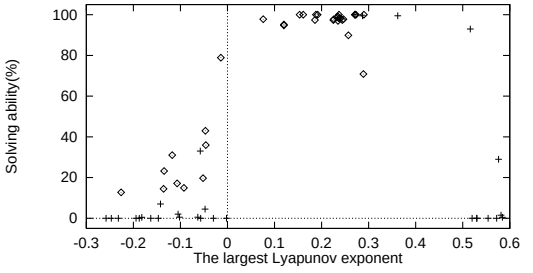

Fig. 5: The relation between solving abilities and the largest Lyapunov exponents. The symbols in this figure are the same as Fig.4.

5. First order statistics of the firing patterns.

The dynamics at the onset of chaos has high complexity and high computational ability [7]. On the edge of chaos, transients becomes very long and behavior of the system exhibits much complicated dynamics until transients vanish. The response on the real edge of chaos, at which the Lyapunov exponents are completely equal to 0, would never converge and complicated behavior continues forever. In this section, as a measure of average "complexity" of responses of chaotic neural networks, we calculate the normalized standard deviation of the firing intervals of neurons. It is defined by the following equation,

$$C_v = \frac{\sigma_a}{\mu_a} \tag{22}$$

where σ_a and μ_a are the standard deviation and the average value of the firing intervals. C_v's are calculated for firing intervals of each neuron and ensemble averages are estimated. The results are shown in Figs.1 (d), 2 (d), 6, 7 and 8. It is clear that there exist the relations between these three quantities, namely, solving abilities, Lyapunov exponents and normalized standard deviations of firing intervals. Higher values of C_v around $k_r = 0.87$ has very high solving ability. On the edge of chaos, the solving ability is very high, and increasing of the summation of positive Lyapunov exponents leads to the low ability and C_v becomes low. From Figs.5 and 8, the ordered phase corresponds to the ranges of negative Lyapunov exponents, and the disordered phase to the larger Lyapunov exponents. The edge of chaos has high C_v and high solving ability.

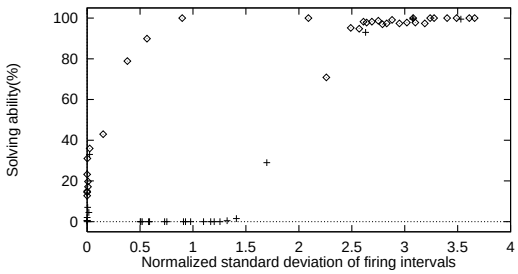

Fig. 6: The relation between solving abilities and the normalized standard deviations of firing intervals. The symbols in this figure are the same as Fig.4.

6. Conclusion.

In this paper, we apply 2-CNN to TSP and show that it is possible to obtain the high solving ability of TSP by changing the decay parameters k_r and k_m, refractoriness and mutual interactions, respectively. The dynamics of chaotic neural networks can be qualitatively described that the larger refractoriness or the smaller mutual interactions, the stronger chaotic response. Our results showed that there are weak chaotic ranges for best searching on finding the optimal solution of TSP. The

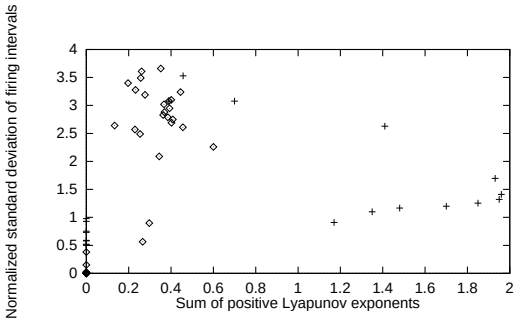

Fig. 7: The relation between sum of Lyapunov exponents and the normalized standard deviations of firing intervals. The symbols in this figure are the same as Fig.4.

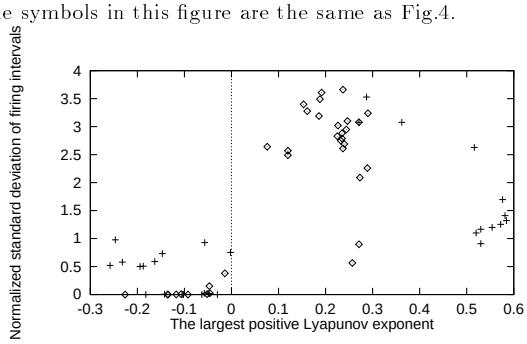

Fig. 8: The relation between the largest Lyapunov exponents and the normalized standard deviations of firing intervals. The symbols in this figure are the same as Fig.4.

high solving ability ranges are observed between two kinds of dynamics, non dynamical and strong dynamical range. High solving ability ranges between these two dynamics can be considered as the edge of chaos. Then, we examine the relation between solving abilities and the Lyapunov exponents. We find typical relations between the parameters k_r and k_m and the Lyapunov exponents. Calculating the normalized standard deviation of the firing intervals as a measure of response of the chaotic neural networks, we find the relation to the solving ability. These results suggest that (i) the edge of chaos has the complicated dynamics and high solving ability, (ii) the neural networks has low solving abilities on the larger Lyapunov exponents, and (iii) the edge of chaos exist between the ordered phase and the disordered phase, which has high solving ability. Finally, it is worth mentioning that we use a kind of first order statistics for measuring the firing patterns in this paper, it is desirable to calculate higher order statistics for evaluating the firing patterns. It is important future problems to explore more effective parameters and to apply the framework of using chaotic dynamics [16] to larger scale optimization problems.

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