Nonlinear Modeling by Radial Basis Function Networks

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Abstract

Deterministic nonlinear prediction is applied to artificial time series data in order to investigate short-term predictabilities, long-term unpredictabilities and orbital-instabilities, which are important characteristics of deterministic chaos. We approximate a functional form of a chaotic neuron model, which is a bimodal mapping, using radial basis function networks, and evaluate performance of approximation by applying nonlinear prediction and estimating Lyapunov exponents. We adopt two methods, the standard and smoothing interpolation: the former is the conventional method that interpolates data points strictly and the latter considers both faithfulness and smoothness of interpolation which is suitable under existence of noise. In order to take a balance between faithfulness and smoothness of interpolation, cross validation is applied to obtain optimal ones. As a result, it is confirmed that by the smoothing interpolation prediction performances are very high and estimated Lyapunov exponents are very similar to actual ones even though in the case of periodic responses.

I. Introduction.

Recently, time series analysis on the theory of deterministic chaos is widely applied to various data, for example, biological data, economical data and so on. Many methods have been proposed for estimating dynamical invariants which characterise deterministic chaos, for instance, fractal dimensions, Lyapunov exponents, and nonlinear prediction. Among them, recent progress of prediction methods enables us to analyse long-term unpredictabilities and short-term predictability, and orbital-instability, the fundamental characteristics of deterministic chaos [1].

On the other hand, there are also many researches in the field of neural computing, for instance, functional approximation, recognition and compression, and so on. One of the popular methods on nonlinear functional approximation is back-propagation learning in neural networks. Although sigmoidal functions are adopted for a basis function in case of back-propagation learning, it is possible to use another functions as a basis function. In this paper, we introduce an approximation technique of nonlinear maps using radial basis function (RBF) networks, which uses a radial function as a basis unit. Concretely, we approximate a nonlinear function which produces chaotic dynamics by evaluating prediction performances and estimating Lyapunov exponents only with time series data obtained from nonlinear maps. As a nonlinear mapping, we analyse a chaotic neuron model which is a bimodal mapping, since it is likely that a bimodal mapping has an important and essential roles in function of our brains [2].

Considering situations similar to real experiments, we use not only the standard radial basis interpolation (RBI) method but also the smoothing RBI method for nonlinear prediction and estimate Lyapunov exponents. With the former method, we analyse a chaotic neuron model without noise, on the other hand, with the latter method, we examine the case under existence of noise. In case of applying the smoothing RBI, it is desired to decide an optimal value of a balancing parameter between smoothness and faithfulness. In this paper, cross validation is applied to solve this problem.

II. Radial basis function networks.

We use RBF networks for approximating and analysing nonlinear maps. We approximate the nonlinear function in accordance with several data points produced from a nonlinear map. The output $\hat{f}(x)$ of this network is given by eq.(1):

$$\hat{f}(x) = \sum_{j=1}^{m} \lambda_j \phi(||x - c_j||), \tag{1}$$

where $x \in R^K$ is an input vector, $c_j \in R^K$ jth RBF center, m is the number of RBF centers and ϕ a radial basis unit, which has a property that the output depends only on the norm between an RBF center and

an input vector for networks. For example, one can utilize Gaussian function as shown in eq.(2):

$$\phi(||x - c_j||) = \exp(-\frac{||x - c_j||^2}{b}), \quad (2)$$

where b is a constant, or thin-plate spline function shown in eq.(3).

$$\phi(||x - c_j||) = ||x - c_j||^2 \log(||x - c_j||), \quad (3)$$

This algorithm is called the standard RBI, which is suitable in case of approximation without noise, because of approximating a functional form on data centers exactly. However in case of approximation under existence of noise, it doesn't necessarily have to interpolate original noisy points. It is important to interpolate as smooth as possible at data centers. Then, we must consider two aspects to calculate λ shown in eq.(1). The first one is to minimize errors $E(\lambda)$ between the original data points and approximated points, and the second one is to interpolate as smooth as possible by minimizing mean curvature rates $C(\lambda)$. These conditions are presented in eqs.(4) and (5):

$$E(\boldsymbol{\lambda}) = \sum_{i=1}^{m} (y_i - f(\boldsymbol{x}_i))^2, \quad (4)$$

$$C(\boldsymbol{\lambda}) = \frac{1}{m} \sum_{i=1}^{m} (\sum_{k=1}^{K} \partial_k \partial_k f(\boldsymbol{x}_i))^2. \quad (5)$$

It is worth nothing that when the one is minimized, the other is large.

Therefore, we should decide a trade-off parameter μ which takes a balance between these relations; in order to evaluate the above issue, eq.(6) is defined,

$$L(\boldsymbol{\lambda}) = E(\boldsymbol{\lambda}) + \mu C(\boldsymbol{\lambda}). \quad (6)$$

In the following, let us define $L(\boldsymbol{\lambda})$ in a form of matrix.

$$\boldsymbol{y} = (y_1, \cdots, y_N)^T, \quad (7)$$

$$\boldsymbol{P} = [P_{ij}] = \phi(||\boldsymbol{x}_i - \boldsymbol{c}_j||), \quad (8)$$

$$\boldsymbol{Q} = [Q_{ij}] = \phi''(||\boldsymbol{x}_i - \boldsymbol{c}_j||) + \phi'(||\boldsymbol{x}_i - \boldsymbol{c}_j||) \frac{K-1}{||\boldsymbol{x}_i - \boldsymbol{c}_j||}. \quad (9)$$

Then, eq.(6) is modified to eq.(10)

$$L(\boldsymbol{\lambda}) = (\boldsymbol{y} - P\boldsymbol{\lambda})^T (\boldsymbol{y} - P\boldsymbol{\lambda}) + \frac{\mu}{N} \boldsymbol{\lambda}^T Q^T Q \boldsymbol{\lambda}. \quad (10)$$

Our aim is to estimate $\boldsymbol{\lambda}$ which minimizes eq.(6). The minimum value of $L(\boldsymbol{\lambda})$ is obtained by setting $\partial L(\boldsymbol{\lambda})/\partial \boldsymbol{\lambda}$=0. Finally, $\boldsymbol{\lambda}$ is calculated as in eq.(11),

$$\boldsymbol{\lambda} = (P^2 + \frac{\mu}{N} Q^2)^{-1} P \boldsymbol{y}. \quad (11)$$

This algorithm is called the smoothing RBI method [3], which is suitable for approximating nonlinear maps under existence of noise. When μ=0, the smoothing RBI is equivalent to the standard RBI method, therefore eq.(11) is modified as follow:

$$\boldsymbol{\lambda} = (P^2)^{-1} P \boldsymbol{y}. \quad (12)$$

Then we ought to solve $P^2 \boldsymbol{\lambda} = P \boldsymbol{y}$.

In general, it is difficult to decide an optimal trade-off parameter μ, since μ depends on a noise level, the number of RBF centers and positions of RBF centers. In this paper, cross validation method [4] is introduced in order to accomplish it. We take the following steps:
(1) The number of data centers m and a value of μ are decided.
(2) A functional form f is approximated in case of removing the ith center, namely with $m-1$ centers. Let us denote the approximated function $\hat{f}_i$ and $f(c_i) = y_i$.
(3) Estimating the map of y_i by $\hat{y}_i = \hat{f}_i(c_i)$, calculate an error $e_i(\mu) = (\hat{f}_i(c_i) - y_i)^2$ and evaluating total square error $E'(\mu) = \sum_{i=1}^{m} e_i(\mu)$.
(4) Computing values of $E'(\mu)$ by changing parameter μ and decide a value of μ at the minimum of $E'(\mu)$ as optimal.

An example with an optimal μ which minimizes mean square error using cross validation method is shown in Fig.1.

Figure 1: An example of the relation between mean square errors and a trade-off parameter μ.

III. Experiments.

A chaotic neuron model was proposed by one of the present authors which qualitatively reproduces various responses including chaos observed in real nerve membranes [2]. In this paper, we approximate a functional form of a chaotic neuron model described by the following one-dimensional mapping:

$$y(t+1) = ky(t) - \alpha f(y(t)) + \mathrm{a}, \quad (13)$$

where $y(t)$ is an internal state of neuron at time t, k is the refractoriness, α and a are parameters, and the output function f is the logistic function, $f(y) = 1/(1+\exp(-y/\varepsilon))$, ε is a steepness parameter.

The functional form of a chaotic neuron model is shown in eq.(13). The bifurcation diagram and the Lyapunov exponents are shown in Fig.2 for parameter a.

At first, we forecast the time series of internal states $y(t)$ of a chaotic neuron model using RBI method. We

Figure 2: The functional form (the top), a bifurcation diagram (the middle) and Lyapunov exponents (the bottom) for the parameter a of a chaotic neuronmodel (k=0.7, ε=0.02, α=1.0).

evaluate prediction performance by calculating correlation coefficients R between actual time series $y(t)$ and predicted time series $\hat{y}(t)$;

$$R = \frac{\frac{1}{T}\sum_{t=1}^{T}(\hat{y}(t)-\hat{y}_a)(y(t)-y_a)}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{y}(t)-\hat{y}_a)^2}\sqrt{\frac{1}{T}\sum_{t=1}^{T}(y(t)-y_a)^2}} \tag{14}$$

where $\hat{y}_a$ and y_a are averages of $\hat{y}(t)$ and $y(t)$, respectively. In this paper, we calculate R with the number of 500 data points.

Secondly, we estimate Lyapunov exponents of a chaotic neuron model using RBF networks. Since the Lyapunov exponents are calculated as shown in eq.(15),

$$L = \lim_{N\to\infty}\frac{1}{N}\sum_{t=0}^{N-1}\log||\frac{df(x(t))}{dx(t)}||, \tag{15}$$

then, the Lyapunov exponents can be estimated by estimating $f'(\boldsymbol{x})$,

$$\hat{f}'(\boldsymbol{x}) = \sum_{j=1}^{m}\lambda_j\phi'(||\boldsymbol{x}-\boldsymbol{c}_j||)\frac{\boldsymbol{x}-\boldsymbol{c}_j}{||\boldsymbol{x}-\boldsymbol{c}_j||}. \tag{16}$$

We show the results that are obtained from an application of both RBI methods to a chaotic neuron model (k=0.7, α=1.0, ε=0.02). In this paper, we show two cases; the first case is periodic responses (a=0.5), the second case is chaotic responses(a=0.35).

IV. Results.

At first, we show the results of approximation without noise using the standard RBI method, the results of nonlinear prediction and estimating Lyapunov exponents, which are shown in Fig.3.

In Fig.3, an approximated map by RBF networks is very similar to the original one. In prediction, correlation coefficients are always high in case of periodic response, on the other hand, they decrease as prediction steps increase in case of chaotic response. Estimated Lyapunov exponents by RBF networks are very similar to actual values not only in case of chaotic response but also periodic response.

When approximating functional forms of maps in case of including noise, it is important to consider how to locate RBF centers. Assuming that RBF centers are extracted from data points produced by a nonlinear map, we locate RBF centers at as regular intervals as possible in existence of noise. The result of approximation, prediction and estimating Lyapunov exponents are shown in Fig.4, where observational noise is added with 20[dB]. In Fig.4, the results using the smoothing RBI method and the standard RBI are compared.

As a result, approximated forms by the smoothing RBI with cross validation is more similar to the original one than by the standard RBI. In prediction, the results considering smoothness have higher precision. The correlation coefficient is decreasing as prediction step increases in a chaotic response, and estimated Lyapunov exponents are almost similar to actual ones both for chaotic and periodic responses. Namely, it is seen that the smoothing RBI algorithm is more effective than the standard one in existence of noise.

V. Conclusion.

In this paper, we approximate a functional form of chaotic neuron models using RBF networks, and evaluate approximated models by applying nonlinear prediction and estimating Lyapunov exponents. In approximation, we consider the case similar to actual experiments; namely, the data is corrupted by noise, especially in case of adopting the smoothing RBI method.

Figure 3: The results of approximating a map (the top), prediction (the middle) and estimating Lyapunov exponents (the bottom) without noise.

Figure 4: The results of approximating a map (the top), prediction (the middle) and estimating Lyapunov exponents (the bottom) in existence of noise.

As a result, we could see that the smoothing RBI is suitable for approximating when noise is included, and that it is effective to apply cross validation to the smoothing RBI algorithm. Although we discuss a one-dimensional case of a chaotic neuron model in this paper, we should extend its application to higher-dimensional dynamical systems, for example, chaotic neural networks, in order to explore possible application of this algorithm to real data. It is also important future problem to consider the optimal number of RBF centers [7] and how to locate them [8].

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