Nonlinear Prediction of Interspike Intervals Produced from Dynamical Systems

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Abstract

Nonlinear prediction is appled to interspike intervals produced from dynamical systems in order to investigate how nonlinear dynamical structures and invariants are transformed. Interspike intervals are produced with the assumption that neuron is modeled by an integrate-and-fire model and driven by dynamical systems. The method of surrogate data is also applied to interspike intervals in order to analyze possible nonlinear deterministic structure that might be included in real time series data.

I. Introduction

It is widely acknowledged that nonlinear prediction can detect existence of chaotic dynamics even if we only observe a single variable time series with relatively small numbers [1]. Another methods, which extract dynamical invariants, such as fractal dimensions and Lyapunov spectra, have several pitfalls on their application to real time series [2].

The method of applying nonlinear prediction to time series data is based on the fact that states of finite dimensional dynamical systems can be reconstructed from the observed time series through the transformation of delay coordinates. This transformation is mathematically guaranteed by the theorem of F. Takens [3] and Embedology by T. Sauer et al. [4]. The effectiveness of this transformation on real time series are confirmed in many fields. For example, the present authors have already applied nonlinear prediction to squid axon data [5, 6] and shown possible existence of chaotic dynamics of squid data. Then effectiveness of applying nonlinear prediction to real time series is suggested in case of existence of deterministic chaos.

In the field of neurobiology, there are different kinds of data series from the above cases. For example, in case of real electrophysiological experiments, it is sometimes the case that measured variables are not continuous quantities, such as membrane potentials, but interspike intervals, or interval time series of successive outputs from neurons.

Recently, T. Sauer has discussed the problems on reconstructing dynamical structure from interspike intervals produced from deterministic dynamical systems [7, 8]. He showed that if a point process is the manifestation of an underlying dynamical system, or the output through a kind of integrate-and-firing models driven by dynamical systems, the states of deterministic systems can be identified from these interspike intervals. This framework has been extended in case of variable thresholds of neurons, which depends on a state of dynamical systems [9].

In this paper, we extend this problem more generally to the case that driving time series are not only deterministic chaos but also stochastic systems. Then, interspike intervals are analyzed by deterministic nonlinear prediction in order to investigate short-term predictabilities and long-term unpredictabilities, the fundamental characteristics of deterministic chaos. Furthermore, the method of surrogate data [10, 11] is adopted for examining nonlinearity of the original interspike intervals.

II. Producing Interspike Intervals from Deterministic and Stochastic Dynamical Systems

Interspike interval time series are made by the following algorithm, which is the same as Refs.[7] and [8]. Let us denote driving time series as $x(t)$ and Θ a positive number corresponding to a threshold. First, a driving time series $x(t)$ is transformed to a positive function $S(t)$, for example $S(t) = \{x(t) + 2\}^2$. Next, the firing times T_i's are defined as the time when integration of $x(t)$ satisfies the following equation:

$$\int_{T_j}^{T_{j+1}} S(t)dt = \Theta. \tag{1}$$

Then the interspike interval time series is made by $t_j = T_{j+1} - T_j$. In deciding the firing timings T_j's, a

linear interpolation is used.

As driving time series, the following cases are analyzed: (i) deterministic chaos, (ii) white noise, (iii) colored noise with the power-law spectra, which is sometimes called fractional Brownian motions, and (iv) noisy periodicity.

Figures 1 – 4 are examples of interspike intervals produced by Eq.(1). In these figures, driving time series $x(t)$, transformed signals $S(t)$ and interspike intervals are plotted in (a) with several statistics, and histograms and reconstructions of the original signals and interspike intervals are drawn in (b); FR, ave, std, cv, N and TH denote a firing rate, an average, a standard deviation, a coefficient of variation and the number of interspike intervals and a threshold Θ, respectively. The coefficient variation C_v is defined as,

$$C_v = \sigma_{t_i}/\bar{t_i} \tag{2}$$

where σ_{t_i} and $\bar{t_i}$ are a standard deviation and an average of interspike intervals t_i. It is often used as an index of relative width of the interspike interval histogram [12]. However, it should be noted here that C_v is just a first order statistics, then a process possessing a different coefficient variation can be realized by the method of surrogate data [10, 11, 13] described in Section IV.

In Fig.1, the driving time series is the variable x of the Lorenz equations,

$$\left\{ \begin{array}{rcl} \dot{x} &=& -sx + sy, \\ \dot{y} &=& (R-y)x - y - xz, \\ \dot{z} &=& -bz + xy, \end{array} \right. \tag{3}$$

where $s = 10, R = 28$ and $b = 8/3$.

Fig.2 shows the case that our neuron is driven by Gaussian random numbers with zero mean and unit variance. In Fig.3 shown are results by a Brownian motion, which is one of the example of fractional Brownian motions. Fig.4 is the result that an input is a sinusoidal time series corrupted by noise. Noise level is 20[dB]. It can be seen that there are qualitative correspondences between the original embeddings and the interspike interval embeddings. In case of driven by deterministic chaos and noisy periodicity, there exists a structure in interspike interval embedding, however, in case of a fractional Brownian motion or random time series, no structures can be clearly seen.

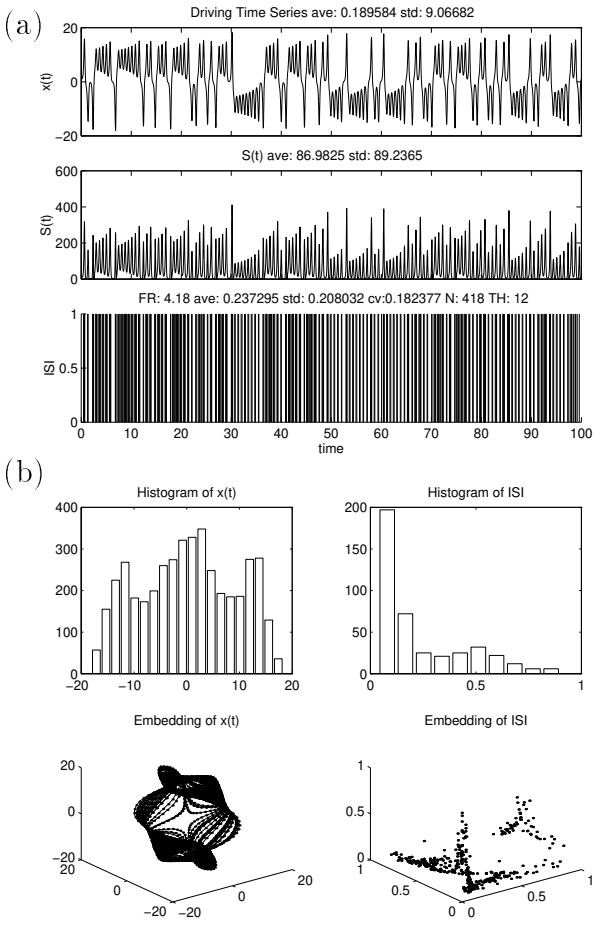

Figure 1: Interspike interval produced from the first variable of the Lorenz equations. (a)Wave forms of driving time series $x(t)$ and transformed time series $S(t)$ are shown in the top and middle panels. In the bottom, produced interspike interval time series is plotted with its statistics. (b)Empirical histograms and three-dimensional embeddings of the driving time series and interspike intervals are shown.

III. An analysis by nonlinear prediction

We use the modified method of analogues [14] as an algorithm of nonlinear prediction, which is very effective even if the analyzed time series are heavily contaminated by additive observational noise [14].

For quantifying prediction performance, two kinds of correlation coefficients are computed, which are called real correlation coefficients and delta correlation coefficients. The first quantity is a conventional correlation coefficient between actual time series $z(t)$ and predicted time series $\hat{z}(t)$ by a nonlinear prediction:

$$r_1 = \frac{\sum_{i=1}^{P}(z(t)-\bar{z})(\hat{z}(t)-\bar{\hat{z}})}{\sqrt{\sum_{i=1}^{P}(z(t)-\bar{z})^2}\sqrt{\sum_{i=1}^{P}(\hat{z}(t)-\bar{\hat{z}})^2}} \tag{4}$$

where $\bar{z}$ and $\bar{\hat{z}}$ are the averages of $z(t)$ and $\hat{z}(t)$, and P is the number of data points in the predicted time series.

The second one is an unconventional measure, which is between difference of actual time series $\Delta z(t) = z(t+1) - z(t)$ and difference of predicted time series

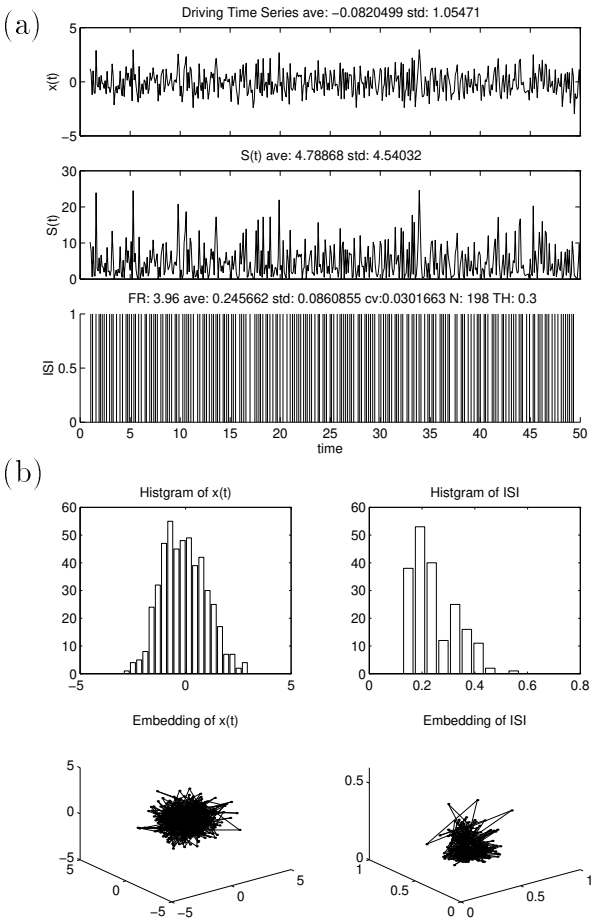

Figure 2: Interspike interval produced from Gaussian random numbers.

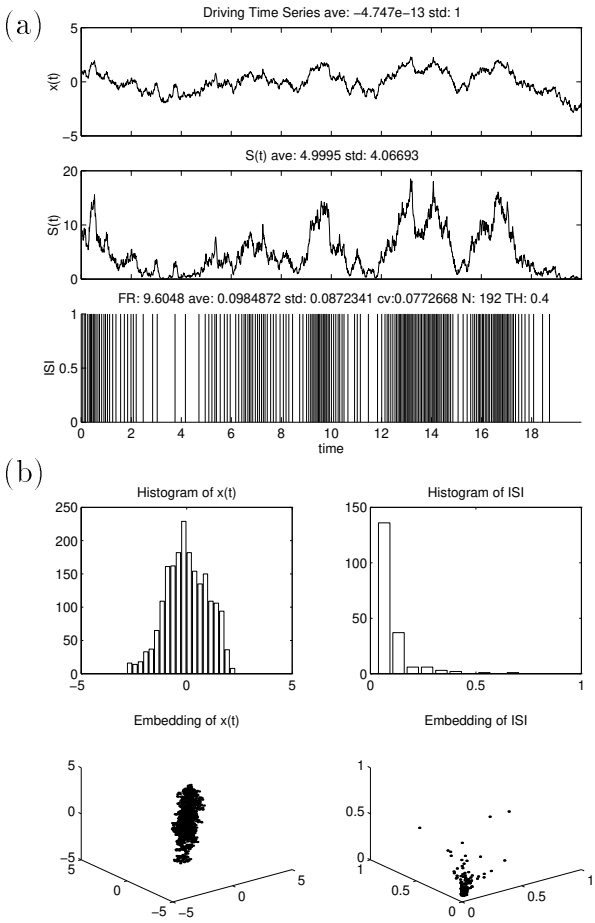

Figure 3: Interspike interval produced from Brownian motions.

$\Delta\hat{z}(t) = \hat{z}(t+1) - z(t)$:

$$r_2 = \frac{\sum_{i=1}^{P}(\Delta z(t) - \Delta\bar{z})(\Delta\hat{z}(t) - \Delta\bar{\hat{z}})}{\sqrt{\sum_{i=1}^{P}(\Delta z(t) - \Delta\bar{z})^2}\sqrt{\sum_{i=1}^{P}(\Delta\hat{z}(t) - \Delta\bar{\hat{z}})^2}}. \quad (5)$$

The latter quantity plays a very important role when nonlinear prediction is applied to detect the possible existence of deterministic chaos [15].

Moreover, it is also important to calculate relative root mean square errors E,

$$E = \sqrt{\sum_{i=1}^{P}(z(t) - \hat{z}(t))^2} \Big/ \sqrt{\sum_{i=1}^{P}(z(t) - \bar{z})^2}, \quad (6)$$

and the signature error S_e defined by the rate of the number of nonidentical signatures between $\Delta z(t)$ and $\Delta\hat{z}(t)$ to the total number,

$$S_e = 1 - \frac{1}{P}\sum_{i=1}^{P} u(\Delta z(t) \cdot \Delta\hat{z}(t)) \quad (7)$$

where $u(\cdot)$ is a step function.

The quantities r_2 and S_e evaluate prediction performance of higher frequency components while real correlation coefficient r_1 and relative root mean square error E evaluate lower frequency components of predicted time series [14]. For evaluating above quantities, time series are divided into two parts. With the first part, nonlinear models are assumed to exist and for the second part, this assumption is tested by applying nonlinear prediction.

Figures 5 and 6 are results of an application of nonlinear prediction algorithm to four time series shown in Figs.1–4.

When neurons are stimulated by deterministic chaos, the Lorenz equations in this case, prediction performance decreases with increasing prediction steps. On the other hand, in case that the input time series is a Brownian motion, there is the same tendency as deterministically driven. Real correlation coefficients have a decreasing tendency with increasing number of prediction steps. However, it is seen that calculating delta correlation coefficients would avoid the misinterpretation that the stochastic time series

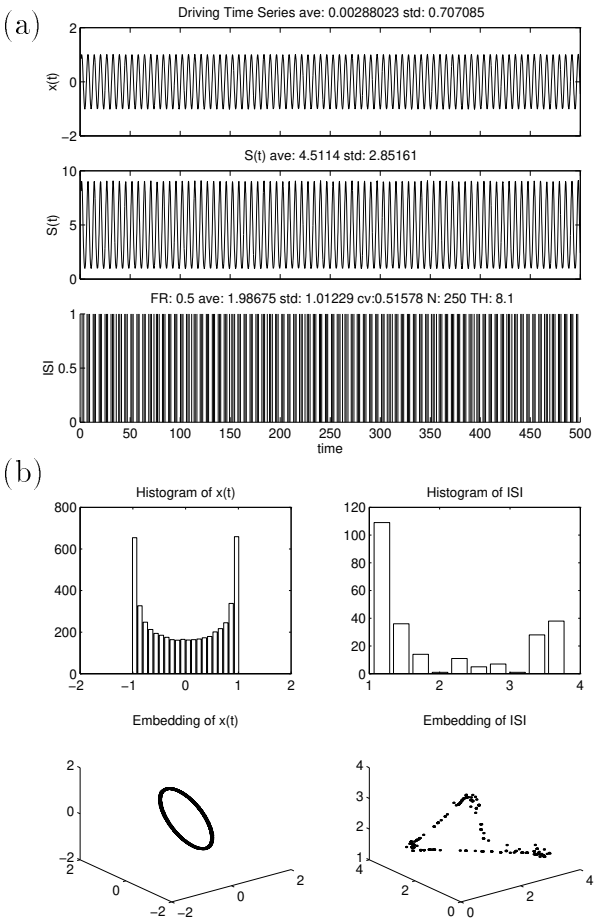

Figure 4: Interspike interval produced from sinusoidal wave corrupted by observational noise.

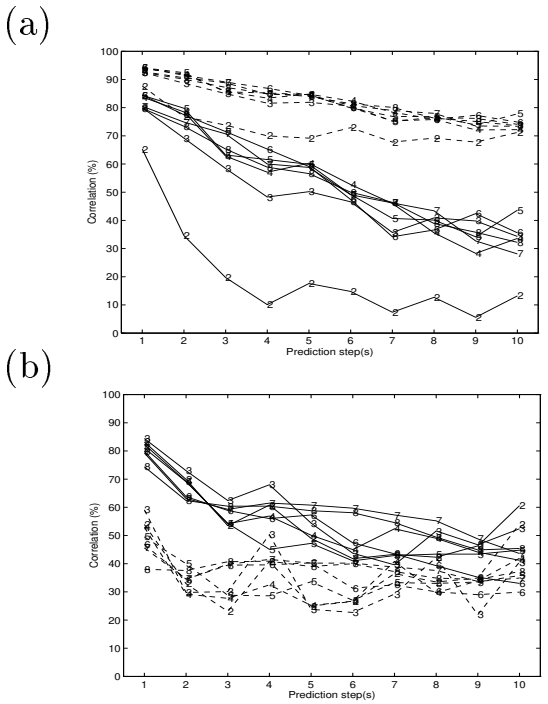

Figure 5: Prediction performance evaluated on examples shown in Figs. 1–4. (a) The Lorenz equation and (b) a Brownian motion. Horizontal axes are prediction steps, and vertical axes are real (solid lines) and delta (dotted lines) correlation coefficients. The numerics in the lines denote reconstructed dimensions m.

with the power-law spectrum is deterministic chaos, which is the same as the case of amplitude reconstruction [15]. For the case of randomly driven, real correlation coefficients are almost zero for any prediction steps. For noisy periodicity, at any prediction steps, prediction is almost possible, therefore prediction performance is very high.

As a result, it is suggested that there are the same tendencies as amplitude reconstruction. Namely, possible existence of dynamical systems behind an integrate-and-fire model can be detected even if only interspike intervals are available.

IV. An analysis on nonlinear determinisity by the method of surrogate data

When time series is analyzed from the view point of deterministic chaos and evidences on existence of deterministic chaos are indicated, it is sometimes claimed that positive results might come from several artifacts induced in experiments or in a process of applying an analysis method. In order to avoid these suggestions, a statistical algorithm, which is called the method of surrogate data [10, 11, 13], is adopted for further studies of exploring nonlinear determinisity in real time series data.

The method of surrogate data assumes existence of a stochastic process against time series as a null hypothesis. We usually consider three null hypotheses; (i) the observed time series has no temporal correlation, (ii) the observed times series can be characterized only by linearity and (iii) the observed time series is an output of an observation function, which is a nonlinear static and monotonic transformation, of linear stochastic time series [10, 11]. The algorithms for making surrogates corresponding to the above null hypotheses are called random-shuffled, phase-randomized and Gaussian-scaled surrogates, respectively.

In order to make random-shuffled surrogates, the data points in time series are randomly shuffled by pseudo random numbers generated by a computer code.

Phase-randomized surrogates are made by preserving the power spectrum and destroying its phase structure. Namely, a surrogate signal made by this algorithm has an identical power spectrum as the original data. It should be noted that it does not preserve empirical histograms.

Although the algorithm for the third null hypothesis is slightly technical, surrogate time series is real-

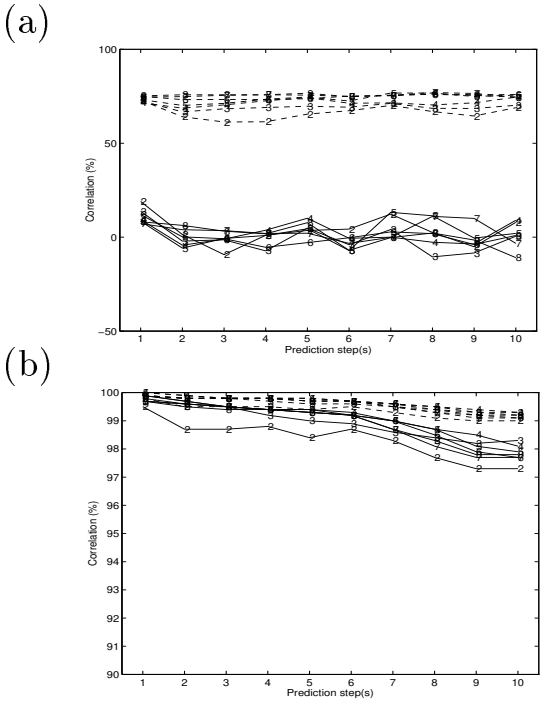

Figure 6: The same as Fig.5 but for (a) white noise and (b) noisy periodicity.

ized by the following steps. Firstly, Gaussian random numbers are produced and then shuffled to follow the rank-order of the original time series. Secondly, the phase-randomized algorithm is applied to this rank-ordered random time series in order to produce surrogate data of the rank-ordered random time series. Lastly, the original time series is shuffled to follow the rank-order of surrogate data produced from the rank-ordered random time series in the previous step. It is worth mentioning that surrogate data made by the third algorithm are shuffled time series of the original data, but this shuffling is a controlled one. Namely, surrogate data sets preserve not only the first-order statistics exactly but also the second-order statistics approximately.

In case of adopting the method of surrogate data to interspike intervals with the second null hypothesis, there is a nuisance problem. Since the phase-randomized surrogate data is made to have an identical power spectrum with the original, a realized surrogate time series has different distributions from the original. For example, in case of interspike intervals, there are only positive value. However, surrogate interspike intervals produced by the phase-randomized algorithm could have negative values. Therefore, a new algorithm is introduced to avoid this difficulty [16]. It is called Fourier shuffled algorithm, which makes surrogate data that preserves the rank-order of the surrogate data produced by the phase-randomized algorithm. Namely, the original time series is shuffled to follow the rank-order of the phase-randomized surrogate data. With this algorithm, autocorrelation is almost preserved.

In order to check whether the null hypotheses can be rejected or not, we adopt Monte Carlo hypothesis testing proposed by Barnard [17] and Hope [18]. If a discriminant statistic Q_0 is observed among the largest $(B+1)\alpha/2$ or the smallest $(B+1)\alpha/2$ in the ranked list of statistics of surrogate data $Q_{H_i}, (i = 1, 2, \ldots, B)$, the null hypothesis is rejected at the level α for a two-sided test [10, 17, 18]. In the following, the number of surrogate data B is fixed to 39 and α to 0.05, then the null hypothesis can be rejected if Q_0 is not included in the tails of an empirical distribution of Q_{H_i}.

In Figs.7 and 8, real correlation coefficient is used as a discrimination statistic. It is clearly seen that for deterministic chaos, real correlation coefficients of the original time series are not included in distributions of those of a surrogate data set. On the other hand, for a Brownian motion, the line of the original cannot be seen because it is included in distribution of surrogates. Namely, the null hypothesis can be rejected for deterministic chaos, but for a Brownian motion it cannot be rejected. The latter property holds for the case of random time series and noisy periodicity.

V. Conclusions

In this paper, nonlinear prediction is applied to interspike interval time series that are produced from deterministic dynamical systems and stochastic dynamical systems. Short-term predictabilities and long-term unpredictabilities are analyzed to detect possible existence of deterministic dynamics including chaos, and distinguish deterministic chaos from stochastics systems. In case of analyzing the original interspike intervals, it is shown that calculating delta correlation coefficients is effective for distinguishing deterministic chaos and fractional Brownian motions. When the method of surrogate data is adopted for further analysis, it is possible to prevent from the misinterpretation that interspike intervals from stochastic systems with a power-law spectrum is produced from deterministic systems, even though only real correlation coefficients are calculated for evaluating prediction performance.

References

[1] G. Sugihara and R. M. May. "Nonlinear forecasting as a way of distinguishing chaos from measurement error in time series," *Nature*, 344, pp.734–741, Apr. 1990.

[2] T. Ikeguchi and K. Aihara. "Surrogate can detect spurious positive Lyapunov exponents of random data," to be submitted Phys. Rev. E, 1995.

[3] F. Takens. "Detecting strange attractors in turbulence," In D. A. Rand and B. S. Young, editors, *"Dynamical Systems of Turbulence"*, *Lecture Notes in Mathematics*, 898, pp.366–381, Berlin, 1981. Springer-Verlag.

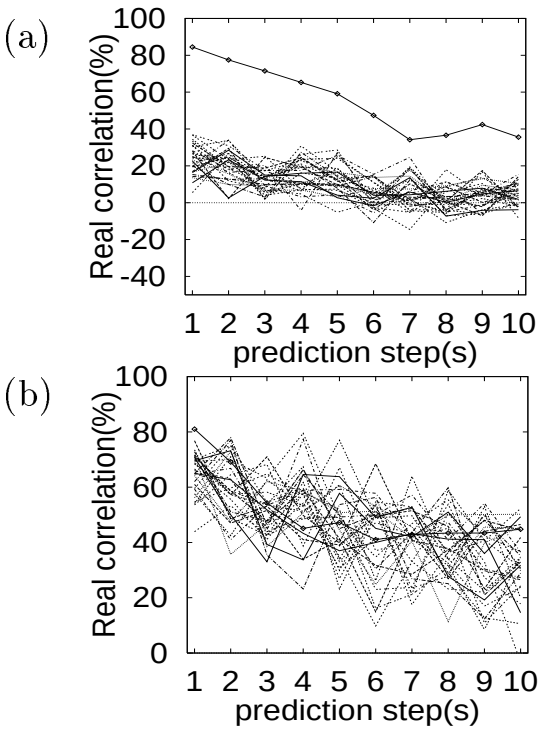

Figure 7: Monte Carlo hypothesis testing applied to the results of application of nonlinear prediction to interspike intervals produced by (a) the Lorenz equation ($m = 6$, Gaussian scaled) and (b) a Brownian motion ($m = 5$, Gaussian scaled). The result of prediction performance for the original interspike interval, real correlation coefficients, is plotted by a solid line with diamonds.

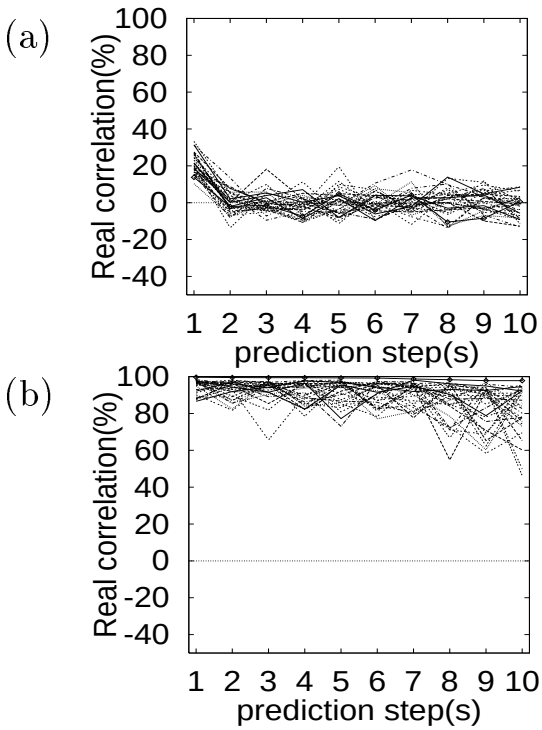

Figure 8: The same as Fig.7 but interspike intervals are produced by (a) white noise ($m = 3$, Gaussian-scaled) and (b) noisy periodicity ($m = 8$, Fourier shuffled).

[4] T. Sauer, J. A. Yorke and M. Casdagli. "Embedology," *J. Stat. Phys.*, 65, 3/4, pp.579–616, 1991.

[5] T. Ikeguchi and K. Aihara. "Nonlinear prediction on squid axon response," Proc. ICNN'95, Perth, 1995.

[6] A. Mees, K. Aihara, M. Adachi, K. Judd, T. Ikeguchi and G. Matsumoto. "Deterministic prediction and chaos in squid axon response," *Phys. Lett. A*, 169, 1&2, pp.41–45, Sep. 1992.

[7] T. Sauer. "Reconstruction of Dynamical Systems from Interspike Intervals," *Phys. Rev. Lett.*, 72, 24, pp.3811–3814, Aug. 1994.

[8] T. Sauer. "Interspike interval embedding of chaotic signals," *Chaos*, 5, 1, 1995.

[9] D. Racicot and A. Longtin. "Reconstructing Dynamics from Neural Spike Trains," Proc. of EMBS 95, 1995.

[10] J. Theiler, B. Galdrikian, A. Longtin, S. Eubank and J. D. Farmer. "Using Surrogate Data to Detect Nonlinearity in Time Series," In M. Casdagli and S. Eubank, editors, *"Nonlinear Modelling and Forecasting"*, pp.163–188. Addison Wesley, 1992.

[11] J. Theiler, S. Eubank, A. Lonting, B. Galdrikian, and J. D. Farmer. "Testing for nonlinearity in time series : the method of surrogate data," *Physica D*, 58, pp.77–94, 1992.

[12] W. R. Softky and C. Koch. "The Highly Irregular Firing of Cortical Cells Is Inconsistent with Temporal Integration of Random EPSPs," *J. Neuroscience*, 13, 1, pp.334–350, 1993.

[13] J. Theiler and D. Prichard. "Constrained-Realization Monte-Carlo method for Hypothesis Testing," Technical Report LA-UR-95-990, Los Alamos National Laboratory, 1995.

[14] T. Ikeguchi and K. Aihara. "Prediction of Chaotic Time Series with Noise," *IEICE Trans.on Fund.*, E78-A, 10, Oct. 1995.

[15] T. Ikeguchi and K. Aihara. "Delta correlation can distinguish deterministic chaos from colored noise," preprint, 1995.

[16] T. Chang, S. J. Schiff, T. Sauer, J.-P. Gossard and R. E. Burke. "Stochastic Versus Deterministic Variability in Simple Neuronal Circuits:I. Monosynaptic Spinal Cord Reflexes". *Biophys. J.*, 67, pp.671–683, Aug. 1994.

[17] G. A. Barnard. "Discussion on The spectral analysis of point process (by M. S. Bartlett)," *J. R. Statist. Soc. B*, 25, p.294, 1963.

[18] A. C. A. Hope. "A Simplified Monte Carlo Significance Test Procedure," *J. R. Statist. Soc. B*, 30, pp.582–598, 1968.