# Non-Periodic Responses of the Izhikevich Neuron Model with Periodic Inputs

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**Abstract**—It is widely acknowledged that neurons show irregular responses with periodic inputs. In this study, we numerically analyzed responses of the Izhikevich neuron model to sinusoidal inputs. Evaluating the diversity index of inter spike intervals (ISIs), we discovered that sinusoidal inputs can produce irregular responses of the Izhikevich neuron model.

## 1. Introduction

It is experimentally shown that real neurons exhibit irregular responses such as chaotic responses or quasi-periodic responses under periodic stimuli such as sinusoidal inputs [1][2]. In Ref. [3], responses of the Izhikevich neuron model [4] to sinusoidal inputs were studied and various mode-locked states were observed. In this study, we used the Izhikevich neuron model with a parameter set of a regular spiking (RS) neuron and analyzed whether sinusoidal inputs can produce non-periodic responses.

## 2. Experiments

In this study, the Izhikevich neuron model were stimulated by a periodic input current defined as $I(t) = I_{DC} + I_{AC} = I_{DC} + A\sin(2\pi/T)t$, where $I_{DC}$ is a constant part and $I_{AC}$ is a periodic part. We set $I_{DC} = 10$ and changed the amplitude $A$ in a range of $0 \leq A \leq 10$ and the period $T$ in a range of $1 \leq T \leq 2000$.

To evaluate the response of the Izhikevich neuron model, we used the diversity index of inter spike intervals (ISIs), which is defined as $R = m/n$ [5], where $m$ and $n$ represent the number of the kinds of ISIs and the total number of ISIs, respectively. In this study, if the $i$th and the $j$th ISIs are equal to two decimal places, these ISIs are defined to be the same. Then, $R \to 0$ means a periodic response and $R \to 1$ implies an irregular, possibly a chaotic response.

Figure 1 shows the diversity indices of ISIs by changing the amplitude and the period of the sinusoidal inputs. The color bar represents the diversity index of ISIs. In Fig. 1, the region where the diversity indices are close to zero corresponds to periodic responses and the region where the diversity indices are close to unity corresponds to irregular responses. When the period $T$ is relatively small, amplitudes less than six can result in periodic responses. However, as $T$ increases, the boundary between periodic responses and irregular responses in Fig. 1 converges to the amplitude $A = 6$.

Figure 2 shows an example of trajectories of the model. Figure 2(a) corresponds to a periodic response and Fig. 2(b) corresponds to an irregular response. In Fig. 2(a), when $A$ is greater than six, the v-nullcline intersects with the u-nullcline if the sinusoidal input takes the minimum value ($I(t) = I_{DC} - A$). In contrast, in Fig. 2(b), when $A$ is less than six, the v-nullcline does not intersect with the u-nullcline. When the amplitude $A = 6$, the v-nullcline is tangent to the u-nullcline at the minimum value of $I(t)$. These characteristics could play a crucial role in producing irregular responses of the Izhikevich neuron model stimulated by the sinusoidal inputs.

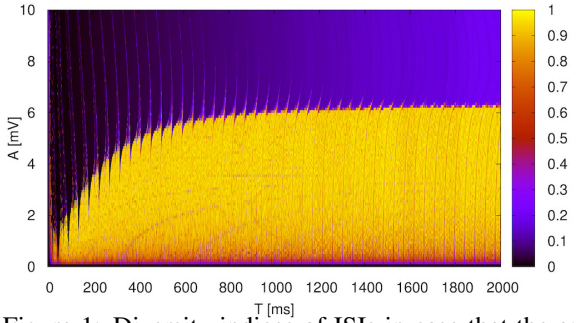
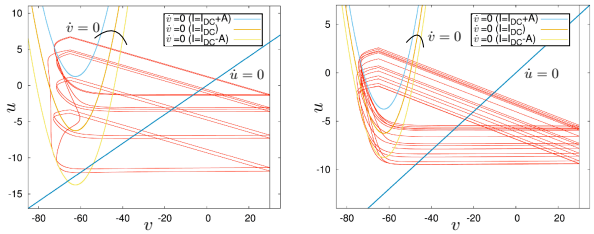



Figure 1: Diversity indices of ISIs in case that the amplitude $A$ and period $T$ changed.




(a) (b)

Figure 2: Trajectories induced by the sinusoidal inputs of (a) a 9-period response ($A = 7.5$, $T = 200$) and (b) an irregular response ($A = 2.5$, $T = 200$).

## 3. Conclusion

In this study, we evaluated responses of the Izhikevich neuron model with sinusoidal inputs by the diversity index of ISIs. As a result of numerical experiments, even if the parameters of the Izhikevich neuron model is set to RS parameters, irregular responses can be observed if the two nullclines do not intersect. The diversity index of ISIs is a simple method for distinguishing periodic responses and irregular responses. Therefore, one of the important future issues is to determine whether irregular responses are chaotic or not in a rigorous way. This research is partially supported by the JSPS Grant-in-Aids for Scientific Research (No. JP17K00348, JP20H000596 and JP21H03514).

### References

[1] Hayashi et al., *PLA*, 88(8), 435–438, 1982.
[2] Aihara et al., *PLA*, 116(7), 313–317, 1986.
[3] Farokhniaee et al., *PRE*, 95(6), 062414, 2017.
[4] Izhikevich, *IEEE TNN*, 14(6), 1569–1572, 2003.
[5] Sugiura et al., *IEICE Tech. Rep.*, NLP2013-34, 2013.